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Surface Temperature Effects on Boundary-Layer Transition

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Introduction

LINEAR stability theory provides guidance on the effects of surface temperature on boundary-layer stability. Cooling the surface stabilizes first-mode disturbances and destabilizes second-mode disturbances.[1,2] Therefore, for subsonic and low supersonic Mach numbers, where first-mode disturbances are the dominant disturbances, cooling the surface should increase the transition Reynolds number. For hypersonic Mach numbers, where second-mode disturbances are the major disturbances, cooling the surface should reduce the transition Reynolds number. Supersonic wind-tunnel transition results have generally been compatible with linear stability theory; however, the hypersonic transition data have produced conflicting results. Reducing the surface temperature in hypersonic transition experiments did not result in a consistent trend. Available data indicate trends of increasing, decreasing, or no change in transition Reynolds number with reductions in surface temperature. Figure 1 contains supersonic and hypersonic wind-tunnel data collected by Potter.[3] $(Re_{x_T})_{AD}$ is the transition Reynolds number obtained under adiabatic conditions, and M_e is the Mach number at the edge of the boundary layer. Surface cooling is seen to significantly increase the transition Reynolds number for the lower supersonic Mach numbers, with a smaller increase at hypersonic Mach numbers. The results of Sanator et al.[4] [not shown in Fig. 1 because the value of $(Re_{x_T})_{AD}$ was not known] at $M_e = 8.8$ indicated no significant change of transition Reynolds number on a sharp cone with changes of wall-to-stagnation temperature ratio from 0.08 to 0.37. Some additional data (not shown in Fig. 1) of Stetson and Rushton[5] at $M_\infty = 5.5$ and Mateer[6] at $M_\infty = 7.4$ indicate a reduction in the transition Reynolds number on cone models with a reduction in the temperature ratio. Thus, the experiments that showed the expected reduction in hypersonic transition Reynolds number with a reduction in surface temperature seemed to be more of an exception rather than the general case.

Contents

Recent boundary-layer stability experiments[7] at a freestream Mach number of 8, along with prior stability experiments of Kendall[8] and Demetriades,[9] have provided a likely explanation for some of the confusion regarding surface temperature effects on hypersonic transition data. Hot-wire anemometry experiments have provided many details of the major disturbances found in laminar boundary layers within a wind-tunnel environment. The major disturbances in the laminar boundary layer of a sharp cone at zero angle of attack at Mach 8 were second-mode disturbances (the high-frequency, acoustical-type disturbances identified by Mack's linear stability analyses[1,2]). Although the planar boundary layer at Mach 8 would also be expected to be dominated by second-mode disturbances, this was not the case.[7] The major disturbances in the laminar planar boundary layer were low-frequency disturbances that were growing in a frequency band that was expected to be stable. These Mach 8 planar results appeared similar to the planar results obtained by Kendall[8] at Mach numbers of 3.0, 4.5, and 5.6 and by Demetriades[9] at $M = 3$. Thus, the instability phenomena producing planar boundary-layer transition in a conventional Mach 8 wind tunnel are different from what was anticipated based on guidance from classical linear stability theory, and it appears that this situation also exists at lower Mach numbers. Some of the confusion associated with hypersonic transition data probably has resulted from the fact it has been incorrectly assumed that hypersonic planar transition was a case of second-mode-dominated transition.

Referring back to Fig. 1, the hypersonic data that had the opposite trend of those expected from stability theory are flat-plate data. Since these are transition data, without any boundary-layer stability information, it is not possible to identify the instability phenomena producing transition. However, in view of the previously mentioned planar instability phenomena, it appears unlikely that second-mode disturbances were significant in these planar laminar boundary layers.

This is one more example of how easy it is to misinterpret transition data. To minimize future misinterpretations, more emphasis should be given to boundary-layer stability experimentation. Through stability experiments a better understanding of instability phenomena and thus a better understanding of transition are obtained.

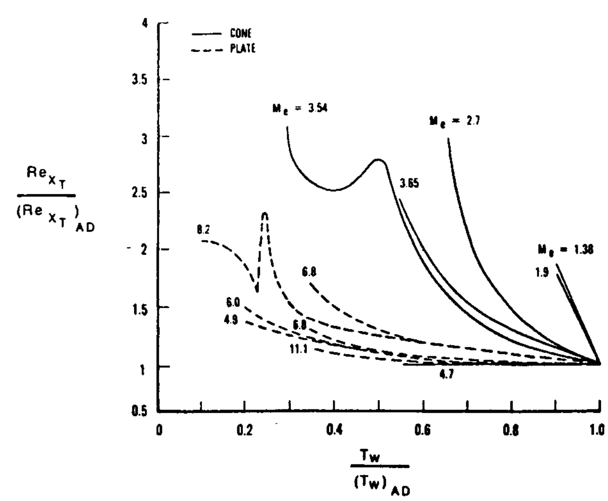

Fig. 1 Effect of surface cooling on transition (from Potter[3]).

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Fully Elliptic Incompressible Flow Calculations on Regular Grid

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Introduction

ONE of the widely used techniques for the solution of the Navier-Stokes equations is the pressure-based relaxation method, where the flowfield is approximated with an assumed velocity and pressure field and updated using a Poisson solver for the pressure.

Caretto et al.[1] developed an original technique that was a breakthrough at the time, but suffered severely from geometrical limitations because the equations were written in Cartesian coordinates, and the staggered grid arrangement did not allow the easy transformation of the equations to a generalized coordinate system. The pressure correction method (PCM) requires the solution of a Poisson pressure correction equation and a subsequent explicit correction of the velocity and pressure field. Rhie and Chow[2] were the first to compute on a regular, or nonstaggered, grid by substituting the full momentum equations into the integral form of the continuity equation. Their solution procedure, however, still relied on the basic pressure correction algorithm.

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Three different substitution formulations with nonstaggered grids have been suggested by Shih and Ren.[3] Some of these employ the Poisson equation for pressure in place of the continuity equation. Their formulation was derived in nonconservative finite difference form in contrast to the derivation of the equations in conservative form in the present study. The code developed in this paper uses pressure weighting to allow the solution of the discretized equations on a regular grid, and the equations are coupled by the substitution of the pressure-weighted form of the momentum equations into the integral form of the continuity equation. The new method permits the direct solution of the pressure, and not the pressure correction, which allows for the additional coupling of the momentum and pressure equations to be solved in block form.

Theoretical Formulation

The equations governing two-dimensional steady incompressible flow are solved in generalized coordinates since the intended application is for the computation of cascade flows. The discretized form of the momentum equations are evaluated at the control volume faces using pressure weighting. This form of the momentum equation is then substituted into the discretized form of the continuity equation which results in an implicit pressure equation. A rigorous derivation of this method is described by Hobson and Lakshminarayana.[4]

The major difference between this derivation and that of Rhie and Chow[2] is evident when one considers the amount of dissipation in the source term of the implicit pressure equation. The present method does not have as much dissipation for the solution of the pressure equation because it only has two pressure gradient expressions in the source term. The Shih and Ren[3] formulation, which is nonconservative, did not include the pressure-weighted method.

The existence of iterative methods with good error-smoothing properties depends upon the ellipticity of the system of equations. Following Shaw and Sivaloganathan,[5] a linearization of the two-dimensional Navier-Stokes equations in Cartesian coordinates is obtained by freezing ρ, μ at ρ_0, μ_0, respectively, and velocities u,v where they contribute to nonlinear terms, at u_0,v_0, respectively. The linearized system may be written as

$$\begin{bmatrix} c-\mu_0\left[\dfrac{\partial^2}{\partial x^2}+\dfrac{\partial^2}{\partial y^2}\right] & 0 & \dfrac{\partial}{\partial x} \\ 0 & c-\mu_0\left[\dfrac{\partial^2}{\partial x^2}+\dfrac{\partial^2}{\partial y^2}\right] & \dfrac{\partial}{\partial y} \\ \dfrac{\partial}{\partial x} & \dfrac{\partial}{\partial y} & 0 \end{bmatrix} \begin{bmatrix} u \\ v \\ p \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \tag{1}$$

where the linearized convective operator is

$$c = \rho_0\left[u_0\frac{\partial}{\partial x} + v_0\frac{\partial}{\partial y}\right] \tag{2}$$

Equation (1) can be written in matrix form as

$$Lq = 0 \tag{3}$$

A Fourier and local mode analysis of this system results in an amplification matrix for this system of equations unique to the discretization scheme. Shaw and Sivaloganathan[5] considered the pressure correction scheme, and Hobson and Lakshminarayana[4] considered the pressure substitution method (PSM).

The distribution of the amplification factor for the PSM and that obtained from the analysis by Shaw and Sivaloganathan[5] for the pressure correction method (PCM)(but for collocated variables) are compared in Table 1. The comparison gives the minimum amplification factors for each method.